Quantum Error Mitigation via Quantum-Noise-Effect Circuit Groups

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Reference: Y. Hama and H. Nishi, arXiv:2205.13907.

Introduction 1: Research and Development of Quantum Computers

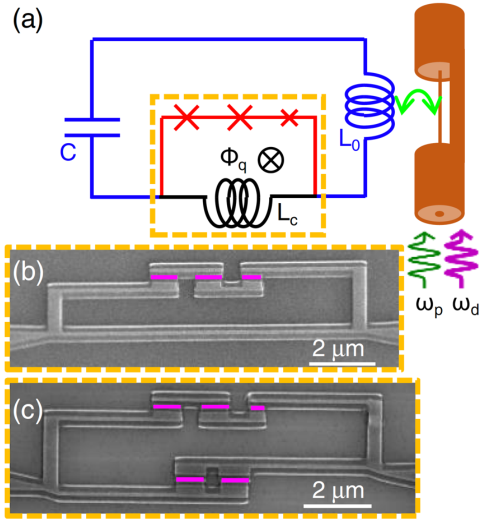
Research and Development of Quantum Computers

- Various types of quantum hardware

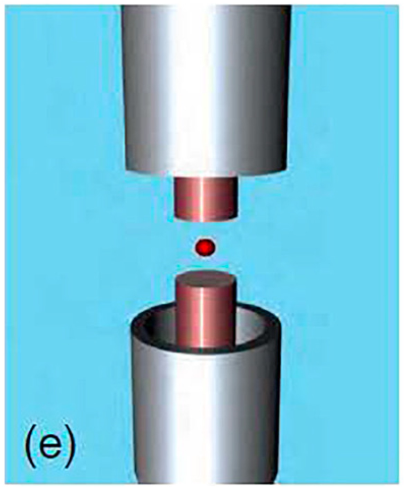
e.g., superconducting circuits, trapped ions, etc.

M. Kjaergaad et al., Appl. Phys. Rev. **6**, 021318 (2019);
Ann. Rev. Conmat. Phys. **11**, 369 (2020).

C. D. Bruzewicz et al., Appl. Phys. Rev. **6**, 021314 (2019).



Superconducting circuits
F. Yoshihara et al., Phys. Rev. Lett. **120**, 183601 (2018).

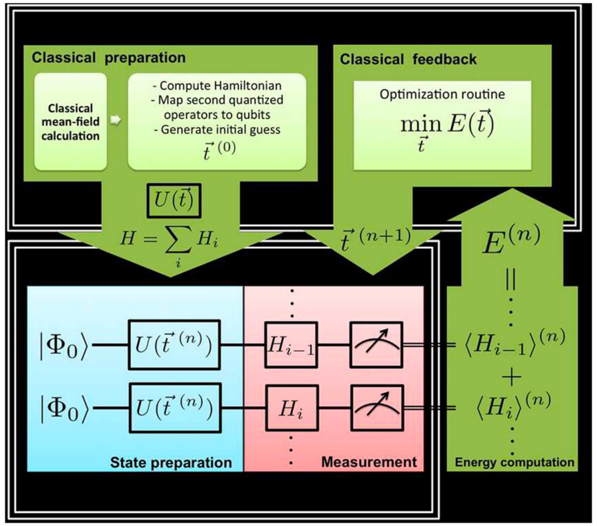


Trapped Ions
C. D. Bruzewicz et al.,
Appl. Phys. Rev. **6**, 021314 (2019).

- Quantum algorithms for material science, quantum chemistry, combinatorial optimization problems

e.g., Variational Quantum Eigensolver (VQE)
Quantum Approximate Optimization Algorithm (QAOA)

A. Peruzzo, et al., Nat. Commun. **5**, 4213 (2014); J. R. McClean et al.,
New. J. Phys. **18** 023023 (2016).
E. Farhi et al., arXiv:1411.4028.



Schematic for VQE implementation
B. Bauer et al., Chem. Rev. 2020, **120**, 12685.

- Demonstration for quantum supremacy with superconducting circuit devices

F. Arute et al., Nature **574**, 505 (2019).

Introduction 2: Quantum Error Mitigation

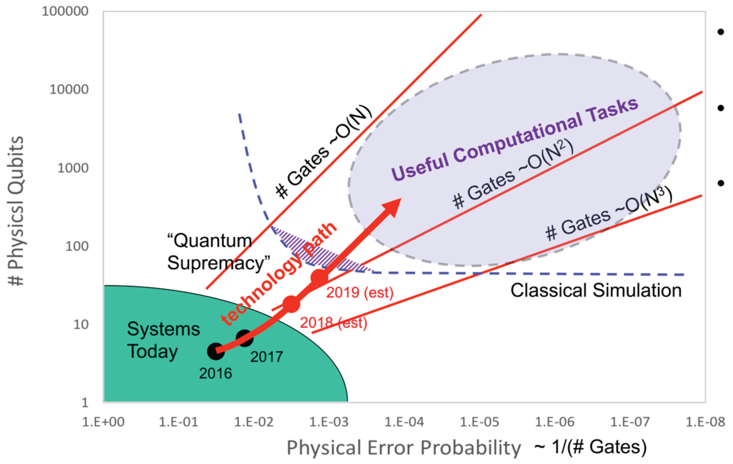
Drawback: Fragile against quantum noise

Tradional error correcting schemes

- Quantum error correcting (QEC) codes

S. J. Devitt et al., Rep. Prog. Phys. **76**, 021318 (2013).

Noisy, Intermediate Scale Quantum (NISQ) Computers



- First, reach a fault-tolerant qubit
- Then scale up in numbers
- Interesting computational tasks beyond classical simulation limit

M. Martonosi and M. Roetteler, arXiv:1903.10541.

problem: QEC codes are not implemented in near-term quantum computers (NISQ devices)

→ necessity for searching alternative error correcting (mitigating) schemes

K. Temme et al., Phys. Rev. Lett. **119**, 180509 (2017).
Y. Li and S. C. Benjamin, Phys. Rev. X **7**, 021050 (2017);
S. Endo et al., Phys. Rev. X **8**, 031027 (2018).
B. Koczor, Phys. Rev. X **11**, 031057 (2021);
New. J. Phys. **23**, (2021) 123047.
W. J. Huggins Phys. Rev. X **11**, 041036 (2021).
Y. Hama and H. Nishi, arXiv:2205.13907.

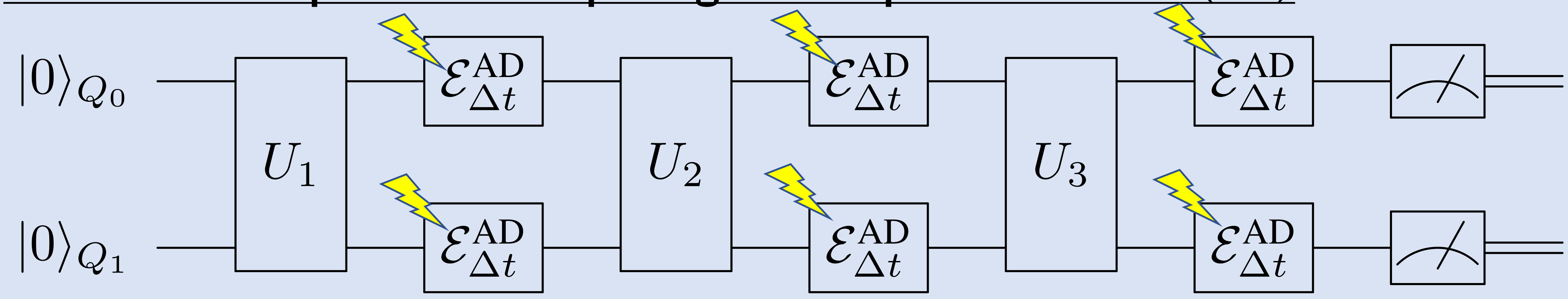
Quantum error mitigation (QEM)

1. zero-noise extrapolation
2. probabilistic error cancellation
3. Exponential suppression derangement/virtual distillation
4. Quantum-Noise-Effect circuit groups

Modeling Noisy Quantum Computation

This work: QEM for quantum computational errors which occur during gate operations, e.g., **amplitude damping (AD).**

Schematic of quantum computing under quantum noise (AD)



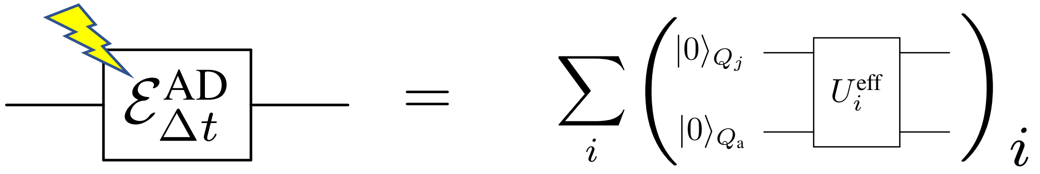
$\mathcal{E}^{\mathrm{AD}}_{\Delta t}$ (Markovian) quantum noise channel during Δt. (Δt : gate operation time)

Goal: Establish **Quantum-Error Mitigation (QEM)** schemes via software

Ideas and strategy for quantum error mitigation (QEM)

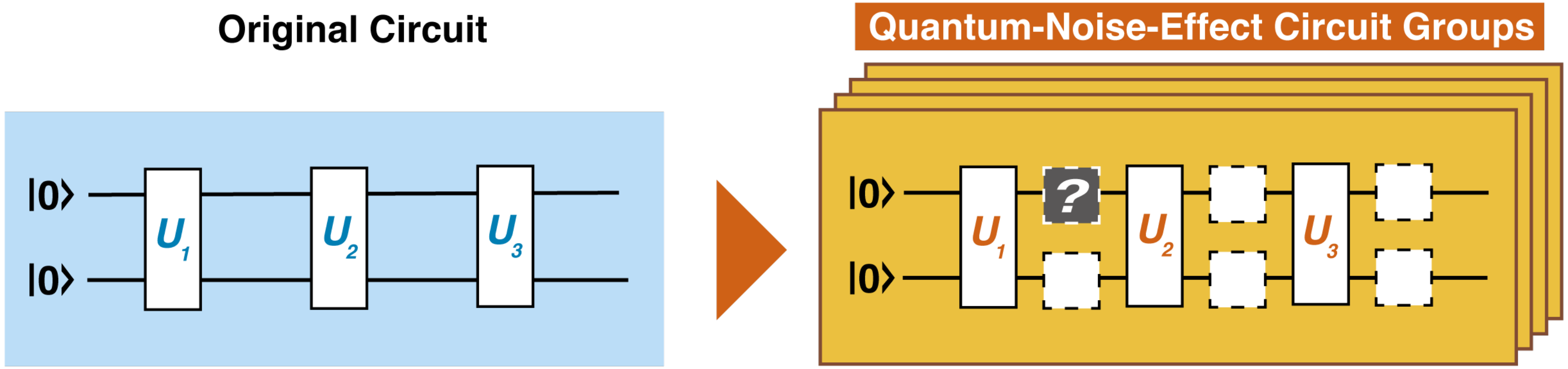
Ideas:

- evaluate quantum noise effect with an ensemble of quantum circuits (quantum-noise-effect-circuit group)



- introduce perturbative expansions for errors characterized by a parameter

$$\tau = \gamma \Delta t \ (\gamma : \text{decay rate})$$



$$\rho^{\mathrm{real}} = \rho^{\mathrm{ideal}} + \tau \cdot \delta^{\mathrm{AD}}_1(\rho^{\mathrm{ideal}}) + \mathcal{O}(\tau^2) \quad (\Delta^{\mathrm{AD}}_1 \rho^{\mathrm{ideal}})^{\mathrm{real}} = \Delta^{\mathrm{AD}}_1 \rho^{\mathrm{ideal}} + \tau \cdot \delta^{\mathrm{AD}}_1(\Delta^{\mathrm{AD}}_1 \rho^{\mathrm{ideal}}) + \mathcal{O}(\tau^2).$$

$$\rho^{\mathrm{real}} - \tau \cdot (\Delta^{\mathrm{AD}}_1 \rho^{\mathrm{ideal}})^{\mathrm{real}} = \rho^{\mathrm{ideal}} + \mathcal{O}(\tau^2). \quad \tau \cdot \Delta^{\mathrm{AD}}_1(\rho^{\mathrm{ideal}}) \approx \tau \cdot \delta^{\mathrm{AD}}_1(\rho^{\mathrm{ideal}})$$

first-order QEM

Applications of our QEM method to QAOA

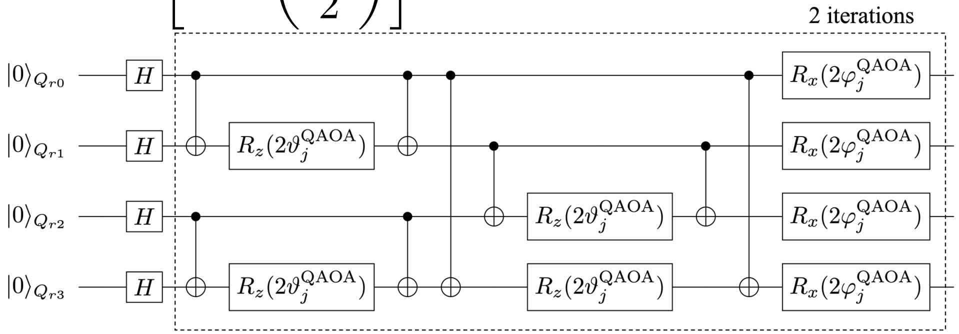
- **QAOA Hamiltonian** $$H_C = \frac{1}{2}\sum_{(i,j)} \left(Z_{Q_{ri}} \otimes Z_{Q_{rj}} - 1\right)$$

- **QEM measure:** $|\mathrm{exp}_{\mathrm{noQEM}} - \mathrm{exp}_{\mathrm{ideal}}| / |\mathrm{exp}_{\mathrm{QEM}} - \mathrm{exp}_{\mathrm{ideal}}|$

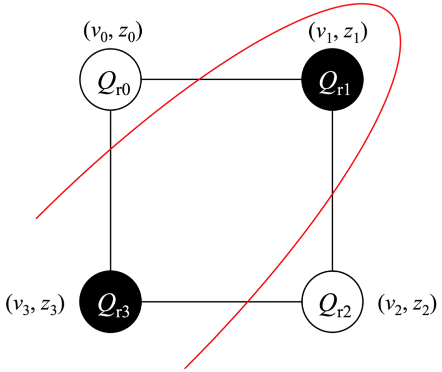
$$\mathrm{RT}_{\mathrm{QEM}} = \frac{|\langle \hat{O}\rangle_{\rho^{\mathrm{real}}_{d\cdots 1}} - \langle \hat{O}\rangle_{\rho_{d\cdots 1}}|}{|\langle \hat{O}\rangle^{\mathrm{QEM}}_{\rho_{d\cdots 1}} - \langle \hat{O}\rangle_{\rho_{d\cdots 1}}|}.$$

$\mathrm{RT}_{\mathrm{QEM}}>1$ implies the validity of our QEM.

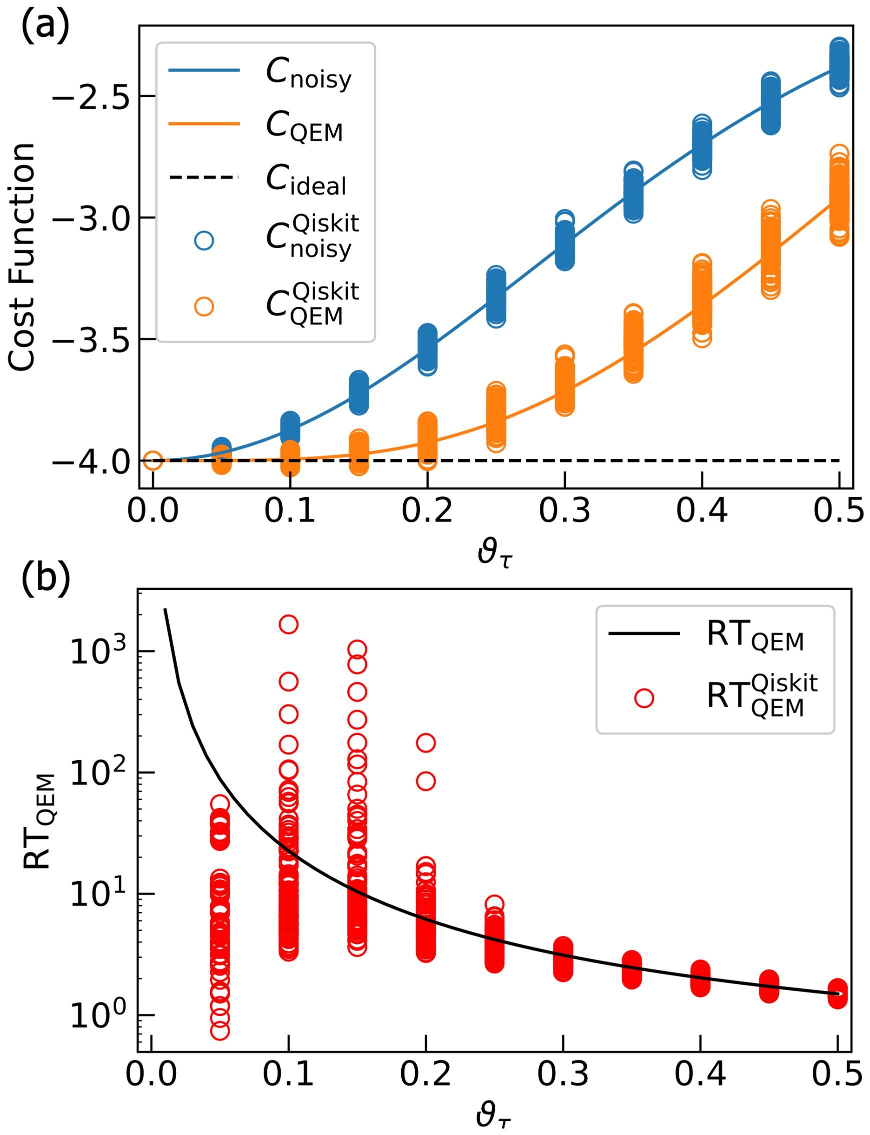
$$\tau = -\log\left[\cos^2\left(\frac{\vartheta_\tau}{2}\right)\right]$$



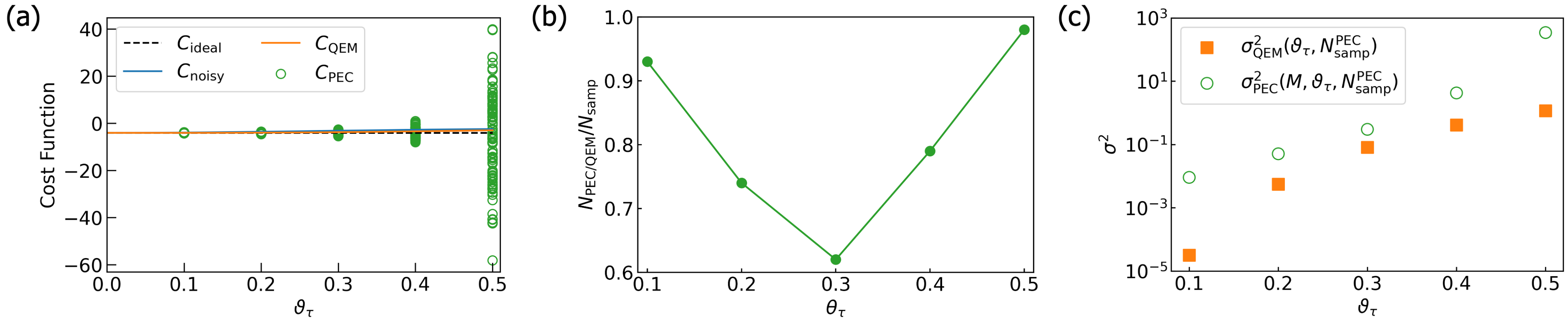
quantum circuit



solid curves: simulation results with our original code
open circles: simulation results with Qiskit code



Comparison with PEC



M=181

$N^{\mathrm{PEC}}_{\mathrm{samp}}$=100

PEC simulation is performed by Cirq and Mitiq

1. Ratio

$$\mathrm{RT}_{\mathrm{PEC/QEM}}(M, \vartheta_\tau, l) = \frac{|\langle \hat{O} \rangle_{\mathrm{PEC}}(M, \vartheta_\tau, l) - \langle \hat{O} \rangle_{\mathrm{ideal}}|}{|\langle \hat{O} \rangle_{\mathrm{QEM}}(\vartheta_\tau) - \langle \hat{O} \rangle_{\mathrm{ideal}}|}.$$

$N_{\mathrm{PEC/QEM}}$: # of PEC data points satisfying $\mathrm{RT}_{\mathrm{PEC/QEM}} > 1$

2. Variances

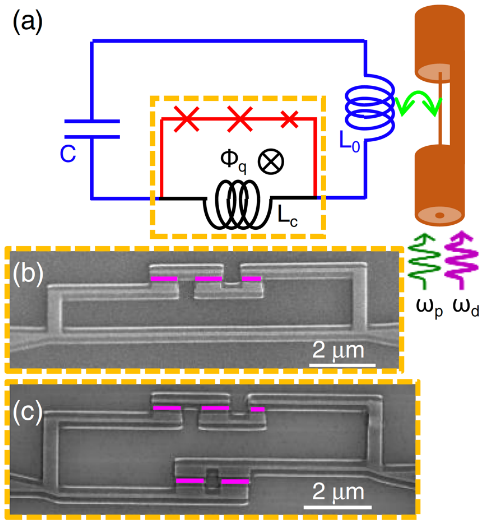
$$\sigma^2_{\mathrm{PEC}}(M, \vartheta_\tau, N^{\mathrm{PEC}}_{\mathrm{samp}}) = \sum_{l=1}^{N^{\mathrm{PEC}}_{\mathrm{samp}}} \frac{1}{N^{\mathrm{PEC}}_{\mathrm{samp}}} \left(\langle \hat{O} \rangle_{\mathrm{PEC}}(M, \vartheta_\tau, l) - \langle \hat{O} \rangle_{\mathrm{ideal}} \right)^2,$$

$$\sigma^2_{\mathrm{QEM}}(\vartheta_\tau, N^{\mathrm{PEC}}_{\mathrm{samp}}) = \sum_{l=1}^{N^{\mathrm{PEC}}_{\mathrm{samp}}} \frac{1}{N^{\mathrm{PEC}}_{\mathrm{samp}}} \left(\langle \hat{O} \rangle_{\mathrm{QEM}}(\vartheta_\tau, l) - \langle \hat{O} \rangle_{\mathrm{ideal}} \right)^2.$$

Result: Our method outperforms PEC.

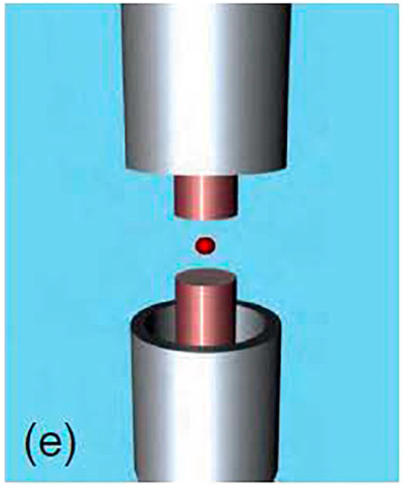
QEM under NISQ parameters

superconducting circuit



Superconducting circuits
F. Yoshihara et al., Phys. Rev. Lett. **120**, 183601 (2018).

trapped ion



Trapped Ions
C. D. Bruzewicz et al., Appl. Phys. Rev. **6**, 021314 (2019).

quantum device	Two-qubit gate time	Decay rate	Perturbation order
superconducting circuits	100 nsec	5 kHz $(1/2T_1)$	1st
trapped ions	100 μsec	0.05 Hz $(1/2T_2)$	1st

Advantages of our quantum error mitigation (QEM) scheme

- It can be implemented with **any type of quantum device**
- It can be applied to **any type of quantum algorithm** under various types of quantum noise channels with an arbitrary strength.
- Computational cost of our QEM is **polynomial** with respect to the product of the depth of a quantum algorithm and the number of qubits.
- Both the errors associated with gate operations and those occurred due to the additional operations for QEM are self-consistently mitigated in our scheme.
- It can be applied to **other quantum technologies**, e.g., quantum metrology: arXiv:2303.01820.